A Summary of Neutrino Physics

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(Dated: May 17, 2024)

In this report we summarize the theoretical aspects of neutrino physics and examine recent experiments that aim to measure their properties. We first look at a mathematical extension of the standard model that yields neutrino masses, show how it can viewed as the limit of a high energy theory, and finally show that it gives rise to the phenomenon of neutrino oscillations. On the experimental side, we describe how different experiments aim to measure the neutrino masses and the mixing angles in the PMNS matrix.

I. INTRODUCTION

Neutrinos are among the most interesting and mysterious particles in the standard model. Part of the reason comes from how light their masses are. It was once thought that neutrinos were massless, and in the conventional formulation of the standard model they do not have masses. Within the standard model, these the neutrinos are defined as the second component of the left handed lepton doublet $L_{Li}(1,2)_{-1}$. This means neutrinos are left handed and come in three flavors, which we label ν_e, ν_μ and ν_τ. Since neutrinos only have left handed components, the field content of the standard model does not allow for a mass term. This implies the existence of an accidental symmetry, namely the conservation of lepton flavor number. It was only after experiments showed neutrinos can change their flavor that this assumption was disproven. Knowing that neutrinos have mass, the next question was how to extend the standard model to accommodate this experimental fact. We will go over different mechanisms by which neutrinos can acquire mass in the standard model and then discuss how modern neutrino experiments measure the parameters related to mass and flavor mixing.

II. NEUTRINO THEORY

A. The Question of Neutrino Mass and the νSM

In the ordinary standard model there is no term we can write that gives masses to the neutrinos [1]. The existence of neutrino oscillations shows that they do indeed have masses. The mass of the neutrino can be explained if we include $d = 5$ non-renormalizable terms in our Lagrangian. The only such term consistent with the symmetries of the standard model is

$$\frac{z_{ij}}{\Lambda}\phi^T\phi L_{Li}^T L_{Lj} \quad L_{Li}^T \equiv \overline{L_{Li}^c} \tag{1}$$

After spontaneous symmetry breaking of the Higgs field, this coupling term gives rise to a Majorana mass term for the neutrinos:

$$\frac{1}{2}(m_\nu)_{ij}\nu_i^T\nu_j \quad (m_\nu)_{ij} = \frac{v^2}{\Lambda}z_{ij}^v \tag{2}$$

where v is the vacuum expectation of the Higgs field and Λ is a dimensional coupling constant with units of mass. We require $\Lambda \gg v$ because non-renormalizability means new physics must come in at high energy scales.

Diagonalizing this mass matrix gives us the three neutrino mass eigenstates $\nu_i, i =\in \{1,2,3\}$ with mass term

$$\frac{1}{2}\sum_{i=1,2,3} m_i\nu_i^T\nu_i. \tag{3}$$

We define the mass splittings $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$ which is useful parameter because it can be measured through neutrino oscillations, which we will discuss in a moment.

B. The PMNS Matrix

The manner in which the mass matrix m_ν is usually diagonalized is via the bi-unitary transformation $V_{\nu L}^* m_\nu V_{\nu L}^\dagger$. The end result is that the neutrino mass eigenstates are linear combinations of the flavor eigenstates $(\nu_e, \nu_\mu, \nu_\tau)$. Now, we recall that the charged lepton masses also came from diagonalizing a mass matrix. Let us denote this matrix by V_{eL}, where it serves an analogous role to $V_{\nu L}$. If we then combine the two to form the unitary matrix $U = V_{eL}V_{\nu L^\dagger}$, we find that the $W^\pm$ interaction terms between the charged leptons and the neutrinos can be written in terms of the mass eigenbasis like so:

$$\mathcal{L}_{W,l} = -\frac{g}{\sqrt{2}}\sum_{l=e,\mu,\tau}\overline{l_L}\not{W}^-\nu_{lL} + h.c. \tag{4}$$

$$= -\frac{g}{\sqrt{2}}\sum_{l=e,\mu,\tau}\overline{l_\alpha}\not{W}^-U_{\alpha i}\nu_i + h.c. \tag{5}$$

The matrix U is called the Potecorvo-Maki-Nakagawa-Sakata (PMNS) matrix and serves to couple different flavors of charged leptons to different flavors of neutrinos via the weak interaction. It is directly analogous to role of the CKM matrix in the strong interaction.

If we start counting the number of degrees of freedom in this matrix, we will find that it contains 6 physical degrees of freedom in total. We can parameterize U by three angles which are typically written as θ_{12}, θ_{13}, and θ_{23}. We also have three phases $e^{-\delta}, e^{i\alpha_1}$, and $e^{i\alpha_2}$. This is different from the CKM matrix which has four parameters. The two extra phases are called Majorana phases.

C. Sterile Neutrinos and the Seesaw Mechanism

The non-renormalizability of this neutrino mass term suggests that it should be the low energy limit of a renormalizable theory. One possibility for this UV-completed theory is attained by introducing a right-handed neutrino singlet to the existing fields of the form

$$N_{Ri}(1,1)_0 \tag{6}$$

This field gives us the following renormalizable terms in the Lagrangian.

$$i\overline{N_{Ri}}\not{\partial}N_{Ri} - \frac{1}{2}M^N_{ij}N^T_{Ri} - Y^\nu_{ij}\overline{L_{Li}}\tilde{\phi}N_{Rj} + h.c. \tag{7}$$

After spontaneous symmetry breaking, we get mass terms for the left and right handed neutrinos.

$$\frac{1}{2}M^N_{ij}N^T_{Ri}N_{Rj} + \frac{Y^v_{ij}}{\sqrt{2}}\overline{v_{Li}}N_{Rj} + h.c. \tag{8}$$

We can rewrite this as a multiplication by a 6×6 matrix

$$\frac{1}{2}\begin{bmatrix}\overline{\nu_L} & \overline{N^c_R}\end{bmatrix}\underbrace{\begin{bmatrix}0 & m_D\\ m_D^T & M^N\end{bmatrix}}_{m_\nu}\begin{bmatrix}\nu^c_L\\ E_R\end{bmatrix} \tag{9}$$

$$m_D = \frac{\nu Y^\nu}{\sqrt{2}} \tag{10}$$

We think of m_D as the Dirac mass term combining the left handed and right handed neutrino components. M^N by contrast would be the Majorana mass term for the right handed neutrinos. It is one of the assumptions that the Majorana mass term is far larger than the Dirac term, $M^N \gg m_D$. In this case, the eigenvalues of m_ν separate into three light masses m_1, m_2, m_3 and three heavy masses M_1, M_2, M_3. It can be shown that the heavy masses have the same order of magnitude which we will refer to by m_N such that $M_n \gg v$. The light masses are then of order $O(v^2/m_N)$ which is much lighter than v. We see that the relationship between the light and heavy masses is one of inverse proportionality. As one set of neutrinos become heavier, the other becomes lighter.

The heavier right handed field is deemed a sterile neutrino. So far there has been inconclusive evidence regarding the existence of these particles.

D. Neutrino Oscillations

When propagating in a vacuum, neutrinos can change their flavor. This is because propagation in free space is governed by the mass eigenstates rather than flavor eigenstates. In particular, if Latin indices denote mass eigenstates and Greek indices refer to flavor, then the time evolution of the mass eigenstate $|v_i(t)\rangle$ is defined by the Schrödinger equation

$$|v_i(t)\rangle = e^{-iE_it}|v_i(0)\rangle. \tag{11}$$

where $E_i \approx p + \frac{m_i^2}{2p}$ is the energy of the neutrino under the ultra-relativistic approximation. The mass and flavor eigenstates are related by the PMNS matrix:

$$|v_\alpha\rangle = U^*_{\alpha i}|v_i\rangle. \tag{12}$$

Hence, the probability of detecting a β from an initial state α as a function of time is

$$|\langle v_\beta|v_\alpha(t)\rangle|^2 = |U_{\beta j}U^*_{\alpha i}e^{-iE_it}\langle v_j|v_i\rangle|^2 \tag{13}$$

$$= \delta_{\alpha\beta} - 4\sum_{j>i}\mathrm{Re}(U_{\alpha i}U^*_{\beta i}U_{\alpha j^*}U_{\beta j})\sin^2\left(\frac{\Delta m^2_{ij}L}{4E}\right) \tag{14}$$

$$+2\sum_{j>i}\mathrm{Im}(U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j})\sin\left(\frac{\Delta m^2_{ij}L}{2E}\right) \tag{15}$$

In this case, the distance a neutrino travels is approximately equal to the time t. We see from this expression that the sin terms cause the different neutrino flavors to contribute sinusoidally to the detected state. This is where the concept of neutrino oscillations arises.

The consequences of neutrino oscillations in fact gave the first hints that neutrinos have mass. In the early days of neutrino detectors, it was observed that the number of neutrinos coming from the sun was much less than expected from theoretical calculations. The solution to this problem was that neutrinos produced inside the sun had sufficient time to oscillate into a random mixture of the three flavors, hence the amount detected at the earth would be a third of the simple estimate.

III. NEUTRINO EXPERIMENTS

We will now describe some experiments that aim to measure the properties of neutrinos. This area has become more popular over the years as the puzzle surrounding the neutrino has captured the attention of physicists. We first turn our attention to some of the experiments studying neutrino oscillations. Otherwise, the list of experiments will be presented in no particular order.

A. T2K

T2K is a neutrino oscillation experiment located in Japan with the with the goal of measuring the mixing angle θ_{12} which is the only angle that had not been measured at the time of the experiment [2]. The setup of the experiment consists of a neutrino source and detector set 295 km apart from each other. The neutrinos are emitted by the J-PARC proton accelerator which produces a beam of τ neutrinos in the direction of the detector. There are, in fact, two detectors with one located next to the neutrino source and the other placed far away. The far detector, called Super-Kamiokande, has the goal of detecting the presence of any e neutrinos that were generated by the oscillations. Finally, the experiment had many other goals as well, for example the precision measurement of Δm_{23}^2 and $\sin^2 2\theta_{23}$ and search for possible sterile neutrinos. After the experiment was completed, its goals were completed, confirming that the θ_{12} mixing angle was nonzero and performing measurements of the other parameters.

B. IceCube

IceCube is a neutrino detector located at the south pole with the purpose of detecting astrophysical neutrinos including those produced by the sun or by the interaction of cosmic rays with the upper atmosphere [3]. These neutrinos are interesting because they may provide clues about the origin of those cosmic rays.

IceCube gets past the difficulty of detecting neutrinos by simply having an incredibly large detection volume. It is essentially a block of ice with a triangular lattice of holes and strings containing spaced-out detectors are lowered into the holes. When a high energy neutrino enters the ice, it may interact with one of the water molecules, releasing a charged particles which emits a photon via Cherenkov radiation. These photons are the way by which the presence of neutrinos is detected.

C. JUNO

JUNO is another neutrino detector located in China. It is located 700 m underground and consists of a 20 kton liquid scintillator that will mainly be used to detect neutrinos mostly from two closely located nuclear power plants but also most other kinds of naturally produced neutrinos [4]. The aim of this experiment is similar but distinct from the previous ones. Its main goal is to determine the neutrino mass hierarchy. Currently, we know the values of $|\Delta m_{13}^2|$ and Δm_{21} from the experiment, but that leaves us with an ambiguity in how the neutrino masses are ordered. The two possible orderings of neutrino masses are

$$m_1^2 < m_2^2 < m_3^2 \quad \text{“Normal order”} \tag{16}$$

$$m_3^2 < m_1^2 < m_2^2 \quad \text{“Inverted order”} \tag{17}$$

The measurement $\overline{\nu_e} \to \overline{\nu_e}$ oscillations will provide enough information to resolve this ambiguity and determine how the masses are ordered. A further result of this experiment will be the precise determination of the quantities $\sin^2\theta_{12}, \Delta m_{12}^2$, and $|\Delta m_{32}^2|$.

D. Hyper-K

The Hyper-Kamiokande, or Hyper-K experiment, is designed to succeed the T2K experiment and Super-K detector mentioned above [5]. Like that experiment, the Hyper-K detector will be placed along the trajectory of the neutrons emitted by J-PARC. The detector is self is a tank containing 258 kton of water surrounded by 20000 and 3600 photomultiplier tubes in the inner and outer layer respectively. Similar to IceCube, the photomultipler tubes sense the Cherenkov radiation emitted as a result of the neutrinos interacting with the water.

This experiment aims to improve upon measurements. It aims to measure the mixing $\sin^2(\theta_{23})$ with a sensitivity of ± 0.017. Another goal is to resolve an ambiguity with this particular mixing angle, specifically which octant it lies in. When the experiment is performed, it should provide information on whether $\theta_{23} < \pi/4$ or $\theta_{23} > \pi/4$. It also aims to measure the CP violating phase δ and the angle $\sin^2(2\theta_{13})$.

E. DUNE

DUNE is a neutrino oscillation experiment in a similar spirit to T2K and Hyper-K [6]. In this experiment τ neutrinos are produced at Fermilab and detected 1300 km away at South Dakota in a similar way to the previously mentioned experiments. The far detector for DUNE contains 70 kton of liquid argon. The goals of the experiment are to measure the CP violating phase δ to a precision of about 10% and the mixing angles θ_{23} and θ_{13} to within a few percent. Additional goals are to constrain the quadrant of θ_{23} and determine the neutrino mass ordering.

Because the experiment is expected to start some time in the 2030's, several of the measurements will already have been performed by some of the other experiments we have discussed. For example, it is expected that the JUNO experiment will have resolved the mass hierarchy before DUNE does and the Hyper-K experiment will have measured the CP violating phase.

IV. CONCLUSION

We see that neutrino physics is a very active area of research being the subject of many experimental physics collaborations. What all the different experiments we have discussed have in common is that the difficult part about doing neutrino physics is simply being able to detect them. As neutrinos only interact via the weak force, their cross section is extremely small and thus neutrino detectors must be very large in size.

There is also the fact that no neutrino experiment performed so far has been able to provide a lower bound on the magnitude of the neutrino masses. Rather, the current best upper bound is $m < 0.08\,\text{eV}$ [7]. For future experiments to detect the neutrino mass directly, the precision of experimental techniques must improve.

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